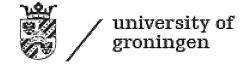
Intergrerend project dynamische systemen

4 April 2014



Exam Problem Sheet

The exam consists of 4 problems. You may answer in Dutch or in English. You have 90 minutes to answer the questions. Give brief but precise answers. You can achieve 50 points in total which includes a bonus of 5 points.

1. [3+5+2 Points.]

Consider the one-dimensional growth model

$$x' = ax - h\sin t$$
,

where a and h are positive real constants.

- (a) Determine the general solution and show that the system has exactly one periodic solution.
- (b) Determine the Poincaré map of the system and show that it has exactly one fixed point which is always a source.
- (c) Use the results in part (b) to sketch the solution curves of the original time continuous system in the (t, x)-plane.

2. [3+3+4+2 Points.]

Let $V: \mathbb{R}^n \to \mathbb{R}$ be a C^{∞} function. Then the system

$$X' = -\nabla V(X) \tag{1}$$

is called a gradient system (here $\nabla V(X) = (\frac{\partial}{\partial x_1} V(X), \dots, \frac{\partial}{\partial x_n} V(X))$ with $X = (x_1, \dots, x_n) \in \mathbb{R}^n$). It is clear that the equilibrium points of a gradient system are given by the critical points of V.

- (a) Show that if X is not an equilibrium point, then V is strictly decreasing along the solution curve through X.
- (b) State the definition of asymptotic stability for the equilibrium point of a time continuous system.
- (c) Show that if X^* is an isolated minimum of V then X^* is asymptotically stable. What can you say about the basin of attraction of X^* in this case?
- (d) What are the conditions on V at an equilibrium point X^* to conclude stability from the linearization of the gradient system at X^* ?

— please turn over —

3. [2+4+5 Points.]

Consider the planar system

$$X' = \begin{pmatrix} 2 & 1 \\ 0 & -1 \end{pmatrix} X.$$

- (a) Determine the canonical form of the system.
- (b) Determine the phase portraits of the original system and its canonical form.
- (c) Determine a conjugacy relating the flows of the system to its canonical form.

4. [3+9 Points.]

- (a) State the definition of chaos for a discrete time system.
- (b) Argue that the doubling map

$$d: [0,1] \to [0,1], \quad x \mapsto 2x \mod 1$$

is chaotic.